**FUNDAMENTALS OF INDUSTRIAL HYGIENE, 6TH ED.**

**HOMEWORK #3**

**INDIVIDUAL SIMPLE AND IDEAL GAS LAWS**

**Objective:** Students will become familiar with the individual ideal gas laws and the combined ideal gas law.

**Background:** The ideal gas law is the equation state of a hypothetical ideal gas. It is a good approximation to the behavior of many gases under many conditions, although it is not without its limitations. The state of an amount of gas is determined by its pressure, volume, and temperature, and the ideal gas law equation relates these in two, very simple, forms. While chemists have long sought an equation that describes the relation of a gas molecule to its environment, they have always encountered difficulties because there are other factors beyond pressure, volume, and temperature (e.g., intermolecular forces) that affect the behavior of certain gases. So again, it is emphasized that this gas law is ideal . . . but to understand the basic concepts of gas behavior, it will be assumed that the gases are in an ideal state and are unaffected by real world conditions. Therefore, for this exercise two well-known assumptions will be made:

1. the particles (atoms, molecules) have no forces acting among them (i.e., intermolecular forces), and

2. the particles do not take up any space (i.e., their atomic volume is completely ignored.

The Ideal Gas Law is simply the combination of all Simple Gas Laws that combines Charles’s law, Boyle’s law, Gay-Lussac’s law, and Avagadro’s law. There is no official founder for this law, rather, it is an amalgamation of the four previously discovered laws. These laws each relate one thermodynamic variable to another mathematically while holding everything else constant.

**Boyle’s Law**

The volume of a given mass of gas is inversely proportional to its pressure (if the temperature remains the same).

$$P\_{1}V\_{1}= P\_{2}V\_{2}$$

where: *P* = pressure (in atmospheres)

 *V* = volume (in liters)

**Charles’ Law**

The volume of a gas is directly proportional to its temperature (if the pressure stays the same).

$$\frac{V\_{1}}{T\_{1}}= \frac{V\_{2}}{T\_{2}}$$

where: *V* = volume (in liters)

 *T* = temperature (in *°K*)

Note: When used in the ideal gas law, temperature must be in degrees Kelvin.

The base for the Kelvin scale is *‘absolute zero’*, the temperature at which all molecular motion ceases.

This temperature is -273 °*C* or -523 °*F*.

$$°C+273= °K$$

or

5/9  (°*F* – 32) + 273 = °*K*

**Gay-Lussac’s Law**

The pressure exerted by a gas is proportional to its temperature (if the volume remains the same).

$$\frac{P\_{1}}{T\_{1}}= \frac{P\_{2}}{T\_{2}}$$

where: *P* = pressure (in atmospheres)

 *T* = temperature (in *°K*)

**Avogadro’s Law**

The volume occupied by a gas is proportional to the number of molecules present in the container.

$$\frac{V\_{1}}{n\_{1}}= \frac{V\_{2}}{n\_{2}}$$

where: *V* = volume (in liters)

 *n* = Avogadro’s number $= \left(\frac{6.022  10^{23}}{mol}\right)$

**Ideal Gas Law**

$$\frac{P\_{1}V\_{1}}{T\_{1}}= \frac{P\_{2}V\_{2}}{T\_{2}}$$

or

$$PV=nRT$$

where: *R* = the *gas constant* of $0.08206  \left(\frac{atm  liter}{mol  °K}\right)$

 If the temperature and pressure are kept constant, then the volume of the gas is directly proportional to the number of molecules of gas.

 If the temperature and volume remain constant, then the pressure of the gas is directly proportional to the number of molecules of gas.

 If the number of gas molecules and the temperature remain constant, then the pressure is inversely proportional to the volume.

 If the temperature changes and the number of gas molecules are kept constant, then either pressure or volume (or both) will change in direct proportion to the temperature.

**Example Problem:**

A DOT 3AA-2015 high-pressure cylinder is used to store oxygen.

The tank measures 9 inches (23 centimeters) in diameter and 51 inches (130 centimeters) high.

Calculate the volume of the cylinder (*in liters*).

*V = πr2h*

where: *V* = volume (in liters)

 *π* = 3.14

 *r* = radius of cylinder (in centimeters)

 *h* = height of cylinder (in centimeters)

$$V=3.14  \left(11.5 cm  11.5 cm\right)  130 cm$$

$$V=3.14  132.25 cm^{2}  130 cm$$

$$V=3.14  17192.5 cm^{3}$$

$$V=53984.5 cm^{3}$$

Conversion: 1000 *cm*3 = 1.0 *L*

$$V=53984.5 cm^{3}  \left(\frac{1.0 L}{1000 cm^{3}}\right) $$

$$V= \frac{53984.5 cm^{3}  L}{1000 cm^{3}} $$

$$V= 54.0 L$$

**Example Problem:**

What is the weight (*in pounds*) of the gas inside the high-pressure cylinder in the above example if it is filled with 305 moles of oxygen (O2).

To determine this, you first need to know the chemical formula of the substance and the atomic weight of each component part (from periodic table).

Water (H2O) consists of two hydrogen atoms bonded to one oxygen atom.

Atomic weights of each element (from the periodic table) are:

$$H= 1.00  2= 2.00$$

$$O=16.00  1=16.00$$

$$ 18.00$$

Answer: One mole of water weighs 18.00 grams.

Atomic weight of oxygen = 16.00

However, free oxygen consists of a diatomic molecule (O2).

Therefore, the weight of one mole of O2 is:

$$O\_{2}=16.00  2=32.00 g$$

And the weight of gas in the cylinder:

$$305 mol  \frac{32 g}{1 mol} = \frac{305 mol  32 g}{mol} = 9760 g$$

Conversion #1: 28.35 *g* = 1 *oz*

$$Weight= 9760 g  \frac{1 oz}{28.35 g}$$

$$Weight= \frac{9760 g  oz}{28.35 g}$$

$$Weight = 344.3 oz$$

Conversion #2: 1 *lb* = 16 *oz*

$$Weight= 344.3 oz  \frac{1 lb}{16 oz}$$

$$Weight= \frac{344.3 oz  lb}{16 oz} $$

$$Weight= \frac{344.3 lb}{16}=21.5 lb$$

**Example Problem:**

What is the internal pressure (*in psi*) exerted on the cylinder when it is filled during the winter when the temperature is 30°*F*?

*PV = nRT*

where: *P* = *unknown*

*V* = 54 *L*

*n* = 305 *mol*

*R* = 0.08206 *atm  L/mol  °K*

*T* = [5/9  (30 - 32) + 273] = [5/9 (-2) + 273] = [-1.1 + 273] = 272 °*K*

$$P  54 L=305 mol  \frac{0.08206 atm  L}{mol  °K}  272 °K$$

$$P  54 L= \frac{305 mol  0.08206 atm  L  272 °K}{mol  °K}$$

$$P  54 L= 305  0.08206 atm  L  272$$

$$P  54 L= 6808 atm  L$$

$$P= \frac{6808 atm  L}{54 L}$$

$$P = 126.1 atm$$

Note: The standard weight of the atmosphere (*atm*), or air pressure, at sea level is equivalent to 760 *mmHg* (22.9 *inHg*) or 14.7 *psi*.

Conversion: 1 *atm* = 14.7 *psi*

$$Pressure= 126.1 atm  \frac{14.7 psi}{1 atm} $$

$$Pressure= \frac{126.1 atm  14.7 psi}{1 atm} $$

$$Pressure = 1854 psi$$

**Example Problem:**

What would the internal pressure (*in psi*) exerted on the cylinder be if it were stored, unused, on a loading dock where it was exposed to the sun and the cylinder temperature reached 100°*F*?

$$\frac{P\_{1}V\_{1}}{T\_{1}}= \frac{P\_{2}V\_{2}}{T\_{2}}$$

where: *P*1 = 1854 *psi*

*V*1 = 54 *L*

*T*1 = 272 °*K*

*P*2 = *unknown*

*V*2 = 54 *L*

*T*2 = [5/9  (100 - 32) + 273] = [5/9  (68) + 273] = [38 + 273] = 311 °*K*

$$\frac{1854 psi  54 L}{272 °K}= \frac{ P\_{2} 54 L}{311 °K}$$

$$\frac{1854 psi  54 L  311 °K}{272 °K}= P\_{2}  54 L$$

$$\frac{1854 psi  54 L  311 }{272  54 L}= P\_{2} $$

$$\frac{576594 psi}{272}= P\_{2} $$

$$2120 psi = P\_{2} $$

***‘or’***

*PV = nRT*

where: *P* = *unknown*

*V* = 54 *L*

*n* = 305 *mol*

*R* = 0.08206 *atm  L/mol  °K*

*T* = [5/9  (100 - 32) + 273] = [5/9  (68) + 273] = [38 + 273] = 311 °*K*

$$P  54 L=305 mol  \frac{0.08206 atm  L}{mol  °K}  311 °K$$

$$P  54 L= \frac{305 mol  0.08206 atm  L  311 °K}{mol  °K}$$

$$P  54 L= 305  0.08206 atm  L  311$$

$$P  54 L= 7784 atm  L$$

$$P= \frac{7784 atm  L}{54 L}$$

$P = 144 atm$

$$P= 144 atm  \frac{14.7 psi}{1 atm} = \frac{144 atm  14.7 psi}{1 atm}= 2118 psi$$

**Example Problem:**

To meet federal regulations and industry standards, high-pressure cylinders must afford a robust safety factor. The manufacturer of the DOT 3AA tank in question tests each tank to ensure it can withstand a test pressure 5/3 times (1.6$\overbar{6}$ X) its rated service pressure. Based on their testing, the tanks produced by this manufacturer are rated at 2015 psi, which means they had been pressure-tested and determined not to fail (rupture) below a minimum pressure of 3360 *psi* (2015 *psi*  1.67 = 3360 *psi*).

**DOT High Pressure Cylinders Pressure Testing Specifications**

DOT Cylinder Maintenance, Retest, and Certification Requirements

 *http://www.c-f-c.com/gaslink/docs/dot\_cylinder.htm*

DOT 3AA (5/3 times service pressure)

DOT 3BN (2 times service pressure)

DOT 4BW (2 times service pressure)

DOT 4E (2 times service pressure)

Note: Manufacturers will often list the rated pressure of their cylinders on the cylinder tag.

For example, the cylinder tag for the example being used would be stamped 3AA-2015.

Unfortunately, the industrial user of this cylinder does not maintain good environmental hygiene at their facility. Windblown leaves and paper litter have accumulated around the cylinder, and a carelessly thrown cigarette butt has set them afire. Flame temperatures are 450 °*F*. If the fire continues to burn and the temperature of the cylinder and its contents reaches 450 °*F*, what will happen?

$$\frac{P\_{1}V\_{1}}{T\_{1}}= \frac{P\_{2}V\_{2}}{T\_{2}}$$

where: *P*1 = 1854 *psi*

*V*1 = 54 *L*

*T*1 = 272 °*K*

*P*2 = *unknown*

*V*2 = 54 *L*

*T*2 = [5/9  (450 - 32) + 273] = [5/9  (419) + 273] = [232 + 273] = 505 °*K*

$$\frac{1854 psi  54 L}{272 °K}= \frac{ P\_{2} 54 L}{505 °K}$$

$$\frac{1854 psi  54 L  505 °K}{272 °K}= P\_{2}  54 L$$

$$\frac{1854 psi  54 L  505}{272 54 L}= P\_{2} $$

$$\frac{936270 psi}{272}= P\_{2} $$

$$3442 psi = P\_{2} (The tank will likely rupture.)$$

**FUNDAMENTALS OF INDUSTRIAL HYGIENE, 6TH ED.**

**HOMEWORK #3**

**INDIVIDUAL SIMPLE AND IDEAL GAS LAWS**

**Name:**

**EXERCISES:** Perform the calculations identified below. Show your work neatly and clearly in a manner similar to the examples provided above (i.e., write the formula, define each variable in the formula, show steps of your calculations, identify any conversion factors used).

After calculating each step, round the answer to the nearest tenth before proceeding to the next step’s calculation.

**Part I: Calculation of Volume**

A DOT 3AA-2265 high-pressure cylinder is used to store nitrogen (N2).

The tank measures 7 inches (18 centimeters) in diameter and 43 inches (109 centimeters) high.

Calculate the volume of the cylinder (*in liters*). *(7 points)*

Formula:

where:  =

 =

 =

 =

Calculations:

$$V=$$

$$V=$$

$$V=$$

$$V=$$

Conversion:

$$V=$$

$$V=$$

$$V=$$

**Part II: Calculation of Weight**

What is the weight (*in pounds*) of the gas in the cylinder if it were filled with 185 moles of nitrogen?

*(5 points)*

Atomic weight of nitrogen =

However, free nitrogen consists of a diatomic molecule (N2).

Therefore, the weight of one mole of N2 is:

The weight of gas in the cylinder is: (show your work)

Conversion #1 *(grams to ounces)*:

*Weight =*

Conversion #2 *(ounces to pounds)*:

*Weight =*

**Part III: Calculation of Pressure**

Calculate the internal pressure (*in psi*) if the cylinder had been filled at 65 °*F*. *(9 points)*

Formula:

where:  =

 =

 =

 =

 =

Calculations:

*P  =*

*P  =*

*P  =*

*P  =*

*P =*

*P =*

Conversion:

Pressure =

**Part IV: Determination of Test Pressure**

Determine the test pressure (*in psi*) for this tank (Note: The rated service pressure is 2265 psi). *(2 points)*

Test pressure equals X rated service pressure.

Calculation:

Do you have any concerns about having this cylinder in your facility or on your job site? Why or why not?

*(1 point)*

**Part V: Recalculation of Pressure**

Recalculate the internal pressure (*in psi*) if the cylinder becomes heated to 110 °*F*. *(7 points)*

Formula:

where:

Calculations:

**Part VI: Calculation of Moles**

DOT regulations prohibit filling a tank more than its rated service pressure at 70 °*F*.

a) What is the maximum number of moles of nitrogen that can be placed in the cylinder at this temperature and not exceed DOT regulations? *(7 points)*

b) What is the maximum weight (*in pounds*) of nitrogen that can be placed in the cylinder at this temperature and not exceed DOT regulations? *(2 points)*